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Velocity spikes in separated flows

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In a recent study related to transition in the wake flows behind circular cylinders held transversely to an air stream, Bloor (1964) has reported the observation of velocity 'spikes' and attributed these to the close proximity to the hot wire of vortex centres on the opposite side of the von Kármân vortex street. Further observations of spikes are reported here, and the characteristics of their distribution indicate that other explanations of their form must be found. Some idealized flows are considered, and it is concluded that observations of spikiness within the hot-wire output may be accountable in terms of large-scale distributions of vorticity within the flow convected past the wire, the distributions being reasonable representations of a separated flow. The observations also provide some evidence that small vortices of Strouhal frequency exist on the inside of the coherent separated shear layer, and this may assist in the understanding of the feed-back mechanism whereby the von Kármân street establishes itself as a self-perpetuating phenomenon.

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1. Introduction

In a recent investigation of some of the characteristics of the separated flow in the immediate downstream region from a stationary circular cylinder held transversely to the mean flow, Bloor (1964) has reported observing 'spikes' for various positions in the flow in the display of the instantaneous signal from a hot-wire anemometer. The hot-wire anemometer used was operated on the constant current mode. The suggestion was made that the spikes are seen in the velocity signal when the vortex centres on the opposite side of the von Kármân street are unusually close to the wire. The spikes observed corresponded to a sudden reduction and restoration of velocity occurring at a time close to that where the minimum value of velocity was observed in the instantaneous signal associated with passage of the von Kármân street across the stationary hot wire. These observations were made with the hot wire supported principally in the outer region of the wake where entrainment of the external flow occurs, and no results were presented explicitly for the vicinity of the mid-section plane of the wake.

In recent experiments in the near-wake of circular cylinders, velocity spikes were seen in positions similar to those reported by Bloor, but the presence of spikes was detected at many other positions in the wake as well. The purpose of this paper is to describe and discuss these velocity spikes.

The results presented below were obtained as part of an investigation of near-wake flows. The experimental programme involved measurements in the wakes of circular lucite cylinders, of $\frac{3}{4}$ and $2\frac{1}{2}$ in. diameter, supported in the test section of the Brown University open-return wind tunnel. Measurements were made with a hot-wire anemometer system operated in the constant temperature mode with linearized output. The experiments included systematic traverses across the wake taken at various distances downstream of the rear stagnation point. Most measurements were made for the Reynolds numbers 10,600 and 53,000. The data collected included hundreds of oscillograph traces. In this work the Strouhal frequency is defined as the frequency of shedding of vortices from one side of the cylinder.

2. Details of spikes

In this work, a spike is regarded as consisting of part of the record of instaneous velocity in which a large rapid change occurs, followed by a similar large change in the opposite direction. A spike is to be distinguished from ordinary velocity fluctuations by the characteristic that the pertinent changes in velocity are very large compared with most of the fluctuations seen at the same position. Spikes may occur fairly regularly in a wake which has evidence of definite periodicity. For a large change in velocity to be seen as a spike, its characteristic time scale cannot be much larger than the minimum discernible period of fluctuation.

At different positions, different spike structures were observed. The simplest form of spike is that where the velocity undergoes a single change in one direction followed immediately by an equally smooth and rapid change in the opposite direction, the second change being roughly the same in magnitude as the first. This is the simple spike. For some spikes it was observed that the leading stroke was not a single, smooth change in velocity, but one in which there was some fluctuation, yet the trace for the recovery for the average velocity following the attainment of the extremum was smooth; such spikes can be described as having staggered leads. For the converse case the spike can be described as having a staggered tail. Some spikes have the appearance of sine waves for one single period of large amplitude and very small time scale; such spikes can be described as full-wave spikes. These types were observed most commonly, though other variations were also seen.

The most prominent spikes found in the complete set of oscillograph traces were those detected in the outer portion of the vortex region of the wake, corresponding to the position in which spikes were found by Bloor. When these spikes first begin to appear on the record, they are seen as simple downward spikes. As the flow progresses downstream, the spikes are seen to develop a staggered tail. The spikes begin their development shortly before the achievement of the minimum in the regular periodic velocity variation associated with the von Kármán street. A trace which exhibits some fairly simple spikes, and also shows their variability from vortex to vortex in the region of major vortex development, is displayed in figure 1.

At small distances downstream from the rear stagnation point of the cylinder at a Reynolds number of 53,000 some spikiness was found near the mid-section and also just below the coherent separated shear layer. Spikes seen in the latter region were of all the forms described. At 0·6 diameters downstream of the rear stagnation point and for the region between 1·5 and 2·0 radii from the mid-plane, simple downward spikes of varying intensity were seen to occur from about 6 msec after the maximum velocity of the Strouhal wave and before the minimum.

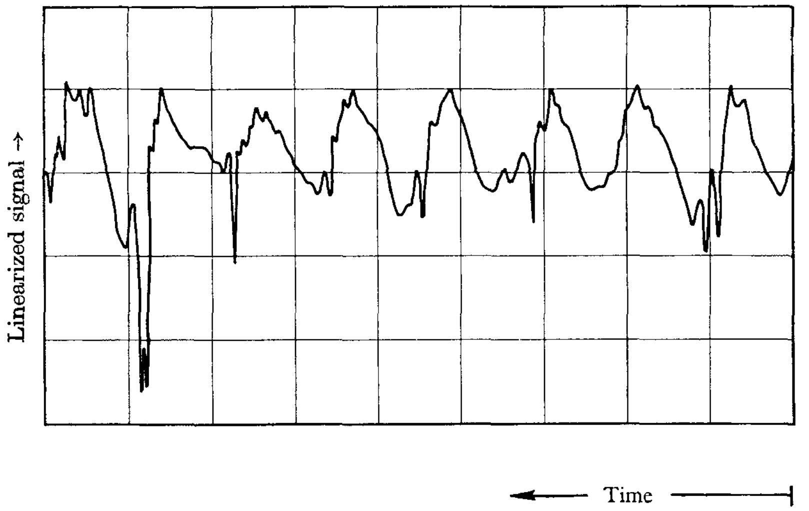

FIGURE 1. Reproduction of typical trace containing spikes. Trace obtained with wire in region of major vortex development. Hot-wire signal has been linearized.

At 0·8 diameters downstream there were simple down spikes and full-wave spikes at 1·4 radii from the mid-plane (and also further out), a few spikes being seen laterally as far away as 2·5 radii. At 1·0 diameters downstream, some full-wave spikes were seen in the vicinity of the mid-plane and simple downward and upward spikes were seen in the region 1·4 radii laterally from the mid-plane.

For a Reynolds number of 10,600 the general distribution of spikes was similar to that at the larger Reynolds number. In the region immediately downstream of the cylinder, and between the two coherent separated shear layers, there was a greater occurrence of moderate spikes, especially full-wave in form. On the other hand, spikes immediately below the coherent shear layers were not as prominent as for the larger Reynolds number. At 0·5 diameters downstream there was little spikiness anywhere across the wake, but at 0·8 diameters and for distances from 1·0 to 1·4 radii from the mid-plane strong evidence of spikes of all forms was seen. By 1·0 diameters downstream the distribution of spikes became different; at 1·0 radii spikes were predominantly simple upwards, at 1·2 the spikes assumed all forms, and from 1·4 upwards the spikes were almost entirely simple and downwards. By 2·0 diameters downstream there is evidence of spikiness of all forms near the mid-plane of the wake, whilst in the region laterally outwards from about 1·4 radii the spikes remain predominantly downwards, showing more frequently a staggered tail. By 3·3 diameters downstream, there is no evidence of the very large downward spikes which have been seen in portions of the wake slightly upstream, but spikiness of smaller amplitude of all types is seen across the wake laterally as far as about 2 cylinder radii. At 12·7 diameters downstream some full-wave spikes were seen in the mid-section of the wake extending laterally outwards to about 0·8 cylinder radii and most forms were seen at positions laterally beyond this range.

It is clear that the form of the spikes should be accountable in terms of the structure of the flow in the vicinity of the hot wire used in measurement. In view of the large number of positions in which spikes are observed, and of the fact that the most prominent spikes do not persist as one traverses transversely to the wake from the outer region in which they have been observed both by Bloor and in the present investigation, towards the mid-plane, it seems that a more detailed explanation of their presence is to be sought other than in the proximity of vortex cores on the opposite side of the vortex street.

3. Idealized flow models for comparison

In order to provide some semi-qualitative comparisons, two flow configurations which idealize certain aspects of the separated flow patterns were considered. The first was a vortex street containing a regular array of vortices of uniform strength, and the second was an isolated starting vortex.

The first idealized flow consisted of the regular von Kármán vortex street with point vortices. The potential for this flow is given by Milne-Thomson (1960). Calculations were made assuming values of vortex spacing typical of the experimental arrangement with appropriate modification of the potential to account for the typically observed velocity and Strouhal frequency of the vortex street. It was found that when the point representing the wire position was sufficiently close to the point-vortex path, large spike-like signals should be expected. When the wire is very close to the vortex paths, the idealized response involves spikes with instantaneous maximum or minimum velocities having magnitudes larger than the average velocity. Also, the spikes would be seen going downwards (for the wire just inside the vortex path) or going upwards (the wire just outside) at times which correspond to positions in the flow where the instantaneous response, which is roughly sinusoidal between the successive vortices, has reached its maximum velocity. None of these features—the large spike magnitudes, the change in spike direction as the hot wire was moved across the average vortex centre path, or the spike origin in the maximum of the Strouhal wave—was observed in the present experiments. This is not surprising, for it is not believed that the model used will provide an adequate description of the distribution of vorticity within a real Kármán street. Equally well, it serves to emphasize that any model chosen for representative study should contain a fairly extensive amount of detail. Unfortunately, this is largely a matter of opening Pandora's box, because there is a large number of periodic vorticity distributions which might be considered seriously as models for the Kármán street, and the effort required in computation increases very considerably as the complexity of the model used also increases. Whilst it is clear that it should be possible to infer a considerable amount of the structural detail of the periodic average vorticity distribution from a set of localized measurements, it would be an extraordinarily complicated computational task to attempt to assess the most probable periodic average vorticity distribution from such a set of data. It is more reasonable to postulate various simplified models for the flow, and to compare the hot-wire response which would be expected as a consequence of such flows with those that are actually observed. An example of this is discussed next.

The second idealized flow considered was that of a vortex developing in the form of a vortex core and an attached curved vortex sheet. In recent years a number of appropriate studies have been made with the aid of digital computers, for example the calculations of Fromm & Harlow (1963) and of Abernathy & Kronauer (1962). These numerical computations are not as amenable for the purposes of computation here as the analysis described by Anton (1956). Anton's analysis related to the case where a flat plate held normal to the direction of its motion is set into motion suddenly with constant velocity in a fluid which was previously completely at rest. At the early part of the motion, vortices are formed at the edges of the plate, and these vortices are initially small compared with the width of the plate. Under these circumstances the vortices from opposite edges interfere but little, so that one can consider the development of each separately, provided that times considered are short.

The analysis outlined here is described with all mathematical details by Anton and his prime references. It is sufficient here to explain the steps taken in the analysis. In essence the analysis consists in transforming the flow from the physical plane to a ζ-plane by the transformation

$$\zeta = (z^2+b^2)^{\frac{1}{2}},$$

where the plane in which the flow takes place is designated as the complex z-plane, and b is the half-width of the plate; the flow in the transformed plane corresponding to the plate flow is composed of the uniform parallel flow and a flow caused by the vorticity of the discontinuity surfaces. This second velocity field is to be calculated. A condition upon the vorticity distribution on the discontinuity surface is obtained by requiring that the velocity should assume finite values at the plate edges. This condition is not, however, sufficient for determination of the distribution density and the shape of the discontinuity surface in space. In order to assess this, it is important to notice first that the development of the discontinuity surface consists initially in a self-similar magnification, since for short times no characteristic length exists to influence the process and the flow must be independent of the magnitude of the vortex at a specific time. This self-similar magnification can be fulfilled only in the case of a certain shape of the discontinuity surface, and of the certain distribution of the vorticity over it. On the basis of this condition, the form and vorticity distribution can be calculated from the beginning of the motion. Using this as a starting condition, one can find, by further computation, the shapes of the vorticity distribution for later times when the vortices from each edge of the plate are no longer small compared with the width of the plate. It can be shown that the laws of self-similar development lead to increase in length scales as the $\frac{2}{3}$ power of time, of circulation as $\frac{1}{3}$ power of time and of velocity as the $-\frac{1}{3}$ power of time. (For demonstration of this, Anton's sources should be consulted.)

For the purpose of computation, it is convenient to regard the vorticity distribution as being concentrated into two regions, the first being a vortex core which contains the rolled-up discontinuity surface within a finite radius, which is attached to and fed by a vortex sheet which is attached at its other end to the plate. For calculation of the velocity field outside the spiral core region, the latter is visualized as replaced by an isolated vortex at the centre of the corresponding circulation; this is permissible since almost circular symmetry prevails in the interior of the spiral region. However, there is asymmetry caused by the use of a separation point to distinguish those parts of the discontinuity surface which lie within the vortex core from those outside, and when a specific assumption for the separation point has been made, the difference between the co-ordinates of the spiral centre and of the centre of circulation can be calculated. Utilizing the essential ideas outlined above, Anton calculated by successive approximations the velocities of the centre of the vortex core and of the points of the outer winding or discontinuity surface which satisfy the laws of similitude.

For the present computations, the velocities at eight points were computed—five points inside the overall spiral and three outside—and for dimensionless times 0·1, 0·2, 0·3, ..., 1·0, 2·0, 3·0, ..., 6·0, at the corresponding similarly situated points. The points were selected along the downstream surface leading from the edge of the plate which is perpendicular to the flow. The dimensionless times were based on the unit time $t_1 = 1{\cdot}42\,H/V_\infty$, determined by Anton, where $2H$ is the body width and V_∞ is the freestream velocity. In determining the velocity components by numerical integration, eighteen points were selected for each outer winding (i.e. discontinuity surface). First, the velocities were computed by numerical integration in the ζ-plane, and then they were transformed to velocities in the physical plane. The resultant velocities were plotted as a function of downstream position for each value of time, and the values of the velocity as a function of time at constant positions were determined by cross-plotting. Typical curves are shown in figure 2.

The curve for each time was found to have the same characteristic form. The characteristic form is such that the velocity reaches a local maximum inside the spiral and then decreases to a minimum from which it again increases inside the spiral after which there is a certain jump in velocity across the discontinuity surface, as would be expected. The velocity near the discontinuity surface, outside the spiral, is a maximum; the velocity outside the spiral then decreases uniformly and reaches the free-stream velocity asymptotically. When the results of the computations are viewed with the velocity as a function of time and constant position, it is found that each curve shows two maxima and a minimum. The first maximum occurs when the discontinuity surface strikes the point under consideration. When the discontinuity surface has passed over the point the minimum is reached in a relatively short time. The second maximum (smaller than the first) occurs when the vortex core comes closest to the point, but beneath it, as would be expected. After the second maximum the velocity reaches the free-stream velocity as time increases. This is not in fact a true result in the case of a plate of finite width since vortices will keep shedding as time goes on and there will also be downstream interferences and interaction between the vortices which started on different sides of the plate. However, it may be emphasized that as the wire position is moved downstream, the final recovery of velocity becomes less rapid.

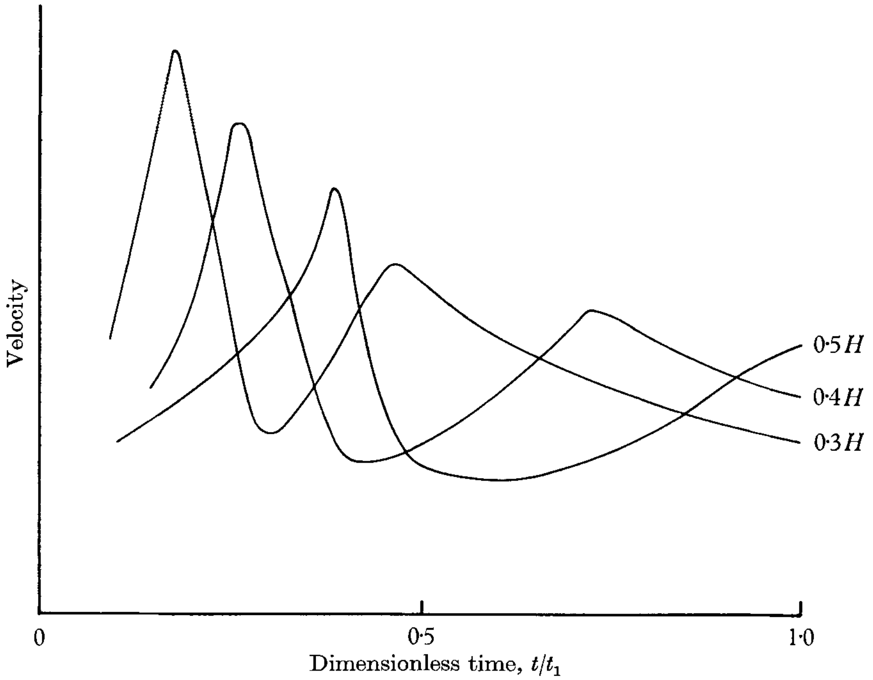

FIGURE 2. Velocity variation with time at various positions downstream from the edge of a plate, from calculations based on Anton's analysis. Plate width is $2H$, and free-stream velocity is V_∞.

4. Discussion

In the present experiments some hot-wire signals somewhat of this latter type were observed. Signals of this type were found inside the separated coherent shear layer, and the feature of less rapid recovery with downstream movement of hot-wire position was observed on spikes in the major vortex formation region. The full-wave spike time scales for experiment and analysis correspond reasonably. This suggests, on the one hand, that the use of a model associated with the starting vortex is useful in so far as it provides a model in which a vortex formation process occurs with the formation rate decreasing in time, and on the other hand, that the observations of spikiness within the instantaneous hot-wire output may be accountable in terms of fairly reasonable distributions of vorticity within the flow convected past the wire. There is probably no great value in attempting to obtain much information from the spike intensity, because the spikes, whilst fairly regular in appearance, did not exhibit great regularity in their magnitude. Further, in converting calculated velocities to expected hot-wire responses it is necessary to assume a specific response of the hot wire to yaw of the flow to the direction normal to the wire axis, and to assign a specific wire orientation in the flow. However, it appears a clear statement in a semi-qualitative sense that the magnitudes as well as the durations of the spikes are accounted for reasonably by such models. The observations also provide positive (though not conclusive) evidence that small vortices of Strouhal frequency exist on the inside of the coherent separated shear layer, for the second idealized flow used in this analysis is somewhat close to the form seen in the pictures of experiments reproduced by Fromm & Harlow (1963). It was found that the Strouhal frequency was strong in this region, in some experiments which are recorded elsewhere. The observation of discrete vortices of Strouhal frequency in this region of the flow which is very close to the point of separation should assist in understanding the feedback mechanism whereby the von Kármán street establishes itself as a stable, self-perpetuating phenomenon.

5. Summary

It has been found that spikes of various forms are observed in the instantaneous velocity signals from hot-wires placed within the region of separated flow behind bluff bodies at relatively small distances downstream of the rear surface. These velocity spikes appear to be accountable both in characteristic period and magnitude in terms of the average periodic vorticity distribution which may reasonably be expected to exist in the regions in which such measurements have been obtained. It also appears that the observations of the distribution of spikes within the near wake are such that there exist discrete vortices of Strouhal frequency on the inside of the separated coherent shear layers, and these may be important as evidence within the chain of processes involved in the establishment of the von Kármán street.

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